Rayleigh-Bénard Convection:
A paradigmatic system for studying pattern formation

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What is pattern formation?

• Many “real” systems are nonequilibrium, due to driving or dissipation

• Many nonequilibrium systems tend to form patterns

• Examples from nature:
  – Snowflakes

photo courtesy of SnowCrystals.com
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photo courtesy of Dr. K. Daniels
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  - Spiral heart waves

photo courtesy of Dr. Bill Ditto, Nature, 392
Examples from the laboratory

• Belousov-Zhabotinsky reactions

photo courtesy of D. Peter Ruoff
Examples from the laboratory

- Belousov-Zhabotinsky reactions
- Granular crispation experiments

photo courtesy of Dr. P. Umbanhower
Examples from the laboratory

- Belousov-Zhabotinsky reactions
- Granular crispation experiments
- Electroconvection

photo courtesy of Dr. M. Dennin
Rayleigh-Bénard Convection

Side View

Overhead View

$T$

$T + \Delta T$

$d$

$g$

Side View

Overhead View
Rotating Rayleigh-Bénard Convection

Overhead View
Rayleigh-Bénard Convection
photos courtesy Dr. G. Ahlers
Goal of Pattern Formation

- Determine if diverse patterns can be understood using reduced equations based on the underlying symmetries.
Stability Diagram
For Rayleigh-Bénard Convection

Spatiotemporal chaos: chaotic in space and time
Transition to Domain Chaos
A debate between experiment and theory

• Theory predicts scaling laws near threshold (Tu and Cross PRL, (1992))
  – How do quantities scale with the control parameter $\epsilon$?
    $\epsilon = \text{constant} \times (\Delta T - \Delta T_c)$
    * precession frequency should scale as $\epsilon^{1.0}$

• Experimenters find (Hu, Ecke and Ahlers, PRL, (1995))
  – Fits to much smaller power laws
    * precesssion frequency should scale as $\epsilon^{0.6}$

• Finite size effects may account for discrepancy
  (Cross, Louie, Meiron, PRE, (2001))
Domain Chaos
Supercomputer solutions of fluid equations with rotation (Coriolis force)
Start with Periodic system
Domain chaos
Supercomputer solutions of fluid equations with rotation (Coriolis force)
Stronger driving (increase $\epsilon$)
Domain chaos
Parameters are exactly the same as for the experiments
Add rigid boundaries
Results for scaling laws: Precession frequency $f$

Theory (Coriolis only) predicts slope $= 1$

Becker, Scheel, PRE, 73

Graph showing
- $f(\text{rad}/\tau_v)$ vs $\varepsilon$
- Slope $= 0.63$
- Slope $= 0.58$
- Slope $= 1.15$
- Symbols: $F_{\text{Cor}} + F_{\text{cent}}$, $F_{\text{Cor}}$, experiment
Centrifugal Force cannot be neglected

- Scaling laws agree better with experiments
- Gives rise to new hybrid state for larger aspect ratio

\[ \epsilon = 0.05, \quad \Omega = 16.25, \quad \Gamma = 80, \quad \sigma = 0.821 \]
Results

Scaling of defect velocities agrees well with theory which predicts $V_{\text{glide}}$ scales as $\epsilon^{0.75}$

Scheel and Cross, PRE, 72
Other work—Lyapunov Exponents

- Chaos exhibits the “Butterfly Effect”—sensitivity to Initial Conditions
  - “Does the Flap of a Butterfly’s Wings in Brazil set off a Tornado in Texas?” —Lorenz, 1972

- Lyapunov Exponent $\lambda$ is a measure of this sensitivity to initial conditions: $\delta u(t) = \delta u(0)e^{\lambda t}$

- Positive Lyapunov Exponent $\rightarrow$ exponential divergence of nearby trajectories $\delta u(0) \rightarrow$ system is chaotic
Chaos

System is truly chaotic in the sense of a positive Lyapunov exponent
\[ \delta u = \text{separation of two initially nearby trajectories}, \quad t = \text{time} \]

Jayaraman, Scheel, Greenside and Fischer, PRE, 74
Visualization of temperature field

color density plot
No rotation, smaller cell
Visualization of temperature field

contour plot
No rotation, smaller cell
Visualization of temperature perturbation field

See spiking in perturbation field during dynamical events

No rotation, smaller cell

Scheel and Cross, accepted by PRE
Conclusions

• The theoretical treatment of scaling laws for the transition to the Domain chaos state needs to incorporate the centrifugal force if it is to accurately model the experimental systems.

• The domain chaos state of rotating Rayleigh-Bénard convection is chaotic in the sense of having a positive Lyapunov exponent.
Future work

• Short Term
  – Incorporate centrifugal force into theoretical formalism
  – Compute all positive Lyapunov exponents, if this scales with system size—gives rigorous definition of spatiotemporal chaos

• Longer Term
  – Continue to study Rayleigh-Bénard Convection
  – Apply dynamical systems approach to other fluid systems such as microfluidics and cardiovascular dynamics.
Results for scaling laws: Correlation Length $\xi$

- For $\xi$:
  - $10^{-2}$
  - $10^{-1}$
  - $5$
  - $10$
  - $15$

- Scaling laws:
  - $\text{slope} = -0.28$
  - $\text{slope} = -0.23$
  - $\text{slope} = -0.39$

- Graph showing $\xi$ vs. some variable $\epsilon$.
- Points and lines indicating experimental data and theoretical fits.
- Graph labels and markers: $F_{\text{Cor}} + F_{\text{cent}}$, $F_{\text{Cor}}$, experiment.