Winning Life: A Study on Using Markov Chains to Find the Optimal Strategy for the Game of Life Board Game

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ABSTRACT

The Game of Life was the country's first popular parlor game. The game simulates a person's journey through their life, from early adulthood to retirement, which includes going through important milestone decisions like college, jobs, marriage, and children. This study models the Game of Life using Markov chains to see, on average, how much a player earns based on which decision they make. Specifically, the study analyzes whether going to college or going straight into a career is the optimal choice to make the most money. The results show that the expected value for money earned when going to college is significantly higher in comparison to going straight to career.

INTRODUCTION

The idea of trying to find the optimal strategy for a game is nothing new. There is an entire branch of mathematics and economics called game theory, which analyzes the interactions between rational decision-makers, typically called players. In the Game of Life, the goal is to optimize the amount of money earned by choosing the decisions that will earn the most money. This research aims to answer the following question: "What is the optimal strategy in playing the Game of Life? On average does going to college earn the player more money than not going to college? "

METHOD

The Game of Life was simplified and modeled where we recorded:

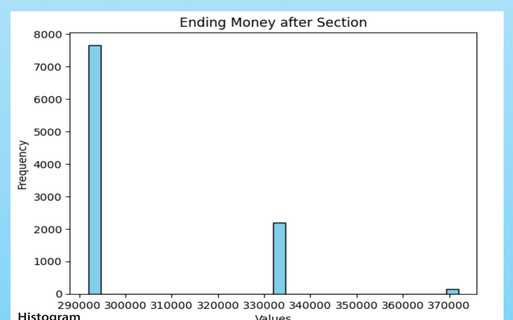
- **The movement of pieces along the board**
- **Money earned through salary spaces**

What we ignored:

- **Event spaces**
- **Friend spaces**
- **Interest spaces**
- **House spaces**

These elements of the game will be added to a later model.

Through the use of properties of Markov chains and simulations, a probability distribution table was found. The table featured represents the probability of ending each section of the board with a specific amount of money. For example if the player chose to go to college, the probability of ending the section with $292,000, $332,000, and $372,000, will be 76.85%, 21.86%, and 1.29% respectively. With this table, we can calculate the expected value and a standard deviation. On average, a player by the end of the section will have $301,776 with a standard deviation of 18222.59. We can then get the Expected Increase of each section where,



PROBABILITY DISTRIBUTION TABLE			
	VALUE	FREQUENCY	PROBABILITY
0	292000	7662	0.7662
1	332000	2202	0.2002
2	372000	136	0.0136

Expected Increase= Expected Amount of Money at the end of the Section - Starting Money, giving us an Expected Increase of $151,776.

By adding all the expected increase of money in each section for a specific path, we can find the expected increase of money for the whole game.

$$\text{Expected Increase}_{\text{Whole Game}} = \sum \text{Expected Increase}_{\text{each section}}$$

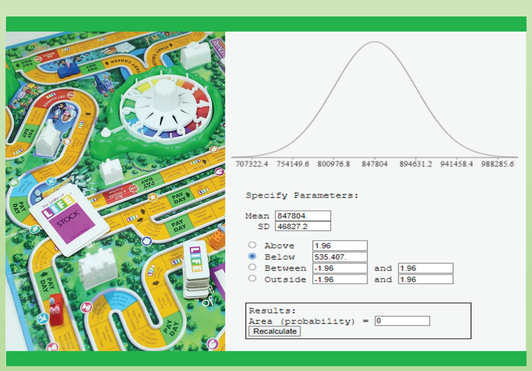
The player would increase money by $847,804.00 if they choose to:

- **Go to college**
- **NOT get married**
- **NOT have a family**
- **NOT have a mid-life crisis**
- **NOT retire early**

in comparison, if the player chose to **NOT go to college** and make the same decisions, he would on average make $535,407.50.
The same method can be used to find the Standard Deviation of the whole game.

$$\text{Standard Deviation}_{\text{Whole Game}} = \sum \text{Standard Deviation}_{\text{each section}}$$

Giving us a standard deviation of $46,827.20 from going to college compared to the standard deviation of not going to college of $17,673.59.



RESULTS

Expected increase of money from going to college is $847,804.00 with a standard deviation of $46,827.20. In comparison to not going to college, it has an expected increase of $535,407.50 with a standard deviation $17,673.59. The amount of money a player can get at the end of the Game of Life is normally distributed.

We can do a one tailed test to see if the expected value of going to college is significantly higher than the expected value of not going to college. We calculated the probability that x= $ 535,407.50 in a normal distribution with a mean of $847,804.00 and standard deviation of $46,827.20. We get that the probability is about 0%, since this is less than our alpha of 5%. This means that the expected value of choosing to go to college is significantly higher than choosing not going to college.

CONCLUSION

In this simplified model of the Game of Life, on average players will earn significantly more going to college rather than opting out of it. The probability of x being greater than $535,407.50 + $100,000 is about 100% in a normal distribution with a mean of $847,804.00 and standard deviation of $46,827.20. This means that on average going to college will earn the player more than $100,000. The initial investment of $100,000 to go to college in the Game of Life is the optimal strategy for the game since the player will earn more than $100,000 than a player who chooses not to get a degree.

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